A Note on:

Why are we teaching students a deeper understanding of how math works?

By
David Smith
Utah State Elementary Math Specialist

Students who understand concepts know why and how mathematical procedures work. For example, when asked to solve a problem involving division of fractions they know more than the old line, "Ours is not to reason why, just invert and multiply". They know that there are two definitions of division, partitive and measurement (also called quotative), and how each or either definition might explain what the problem means. They know why the quotient of a fraction division problem can be larger than the dividend. They can represent the problem in multiple ways. For example, they may use pictures or objects, fraction circles or bars, the number line, a bar model, or a diagram using sets. They can apply their knowledge to solve actual problems involving dividing fractions and can create an application on their own where fraction division is necessary. They can tell you why the inverting and multiplying algorithm works by citing the relationship of division and multiplication and using properties of those operations (the inverse property, in this case) and can explain or justify their reasoning.

Most of us learned mathematics from a very procedural point of view. Our teacher gave examples of the steps one takes to solve a fraction division problem. Then a few of us came to the board and worked two or three other examples, or we all did the problem on an individual chalkboard (today they use whiteboards or iPads). Then we were given a problem set, usually around 30 to 40 problems to solve. We were told to remember the saying, "just invert and multiply." We would work alone in silence or take the problems home for homework. Then we would get to the story problems and, if they were assigned at all, they would throw us for a loop. We had no idea how to interpret them or what to do. Then, the next day we would all correct our papers by handing them to the person behind us while the teacher gave the answers. We didn't even discuss the right answers or the wrong answers. They were just right or wrong and we didn't know why. Then we might have even called out our scores aloud while the teacher recorded them.

One of the many flaws in the traditional "sit and get" approach is that algorithms were taught without understanding. The emphasis was on getting the right answers. A great deal of research in the last 20 to 30 years, however, has indicated that instructional programs that "emphasize understanding algorithms before using them have been shown to lead to increases in both conceptual and procedural knowledge" (Fuson and Briars, 1990; Fuson, Hiebert, Murray, Human, Olivier, Carpenter, Fennemma, 1997; Hiebert and Wearne, 1996).

A classroom in which conceptual understanding, procedural skill and fluency, and application are the goals looks much different. The students may begin to develop their understanding of the concept through a problem solving approach. Students are given a mathematical task to solve the teacher has carefully selected that has a sound mathematical purpose. A task is a problem in context that has some kind of intriguing or even perplexing quality. Students use mathematical concepts they already know to strategize and find solutions to the task. As they do so they may use concrete objects, pictures, drawing, diagrams, numbers, symbols, and so on. Students are encouraged to explore and to talk with each other about what they are doing, to share strategies and work together toward a solution. They are told that they must be able to explain to other students and to the teacher in mathematical language what they did and why they did it. As they are working the teacher is fully involved and circulates around the class instructing, giving suggestions, and taking notes on how the students are working. She asks questions such as:

How else could you have…?
How are these _____________ the same?
How are these different?
About how long…? (many, tall, wide, heavy, big, more, less, etc.)
What would you do if…?
What would happen if…?
What else could you have done?
If I do this, what will happen?
Is there any other way you could…?
Why did you…?
How did you…?

Students may then share what they did and their solution to the problem with others. They tell what they did and why. They show their representations, physical or pictorial or numerical, and defend their reasoning in front of the class or in small groups. The teacher pays close attention to the strategies they use and to how they are leading to understanding the concept. She can then plan for the next step in her instruction.